ESE 415: Optimization

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WUSTL - Spring 2018

Today we will talk about

- **Motivating example**
  Optimization as a pillar of modern imaging
- **Introduction**
  Overview of optimization
- **Class structure**
  Assignments, grades, TAs, etc.

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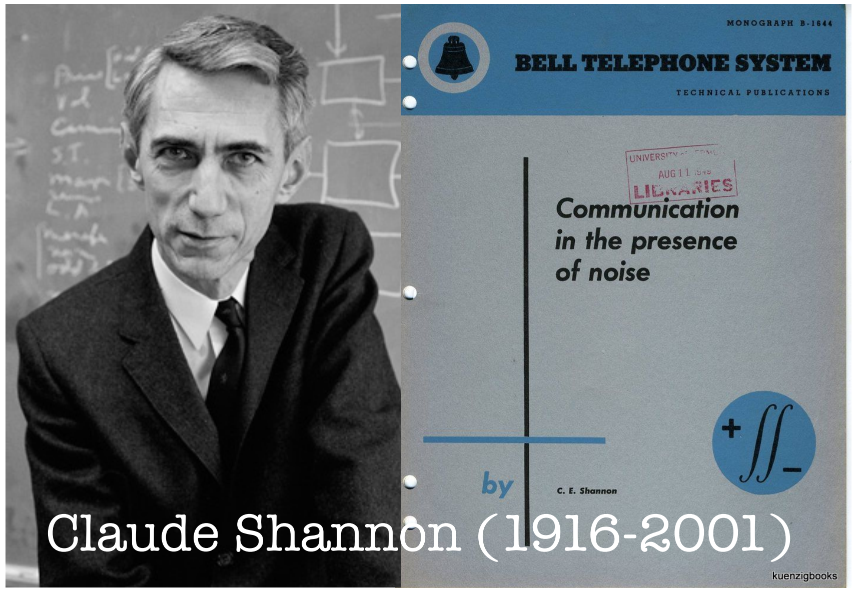
In 1949, Shannon set the foundation of digital era

Theorem [Shannon]: If a function $f(x)$ contains no frequencies higher than ω_{max} (in radians per second), it is completely determined by giving its ordinates at a series of points spaced $T = \pi/\omega_{\text{max}}$ seconds apart.



Claude Shannon (1916-2001)

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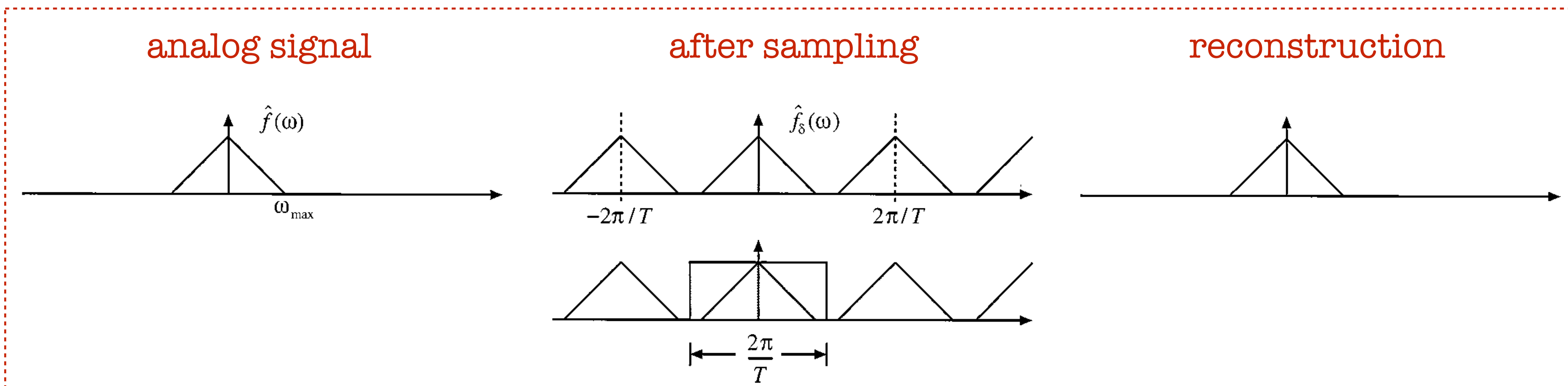
Visual representation in Fourier domain



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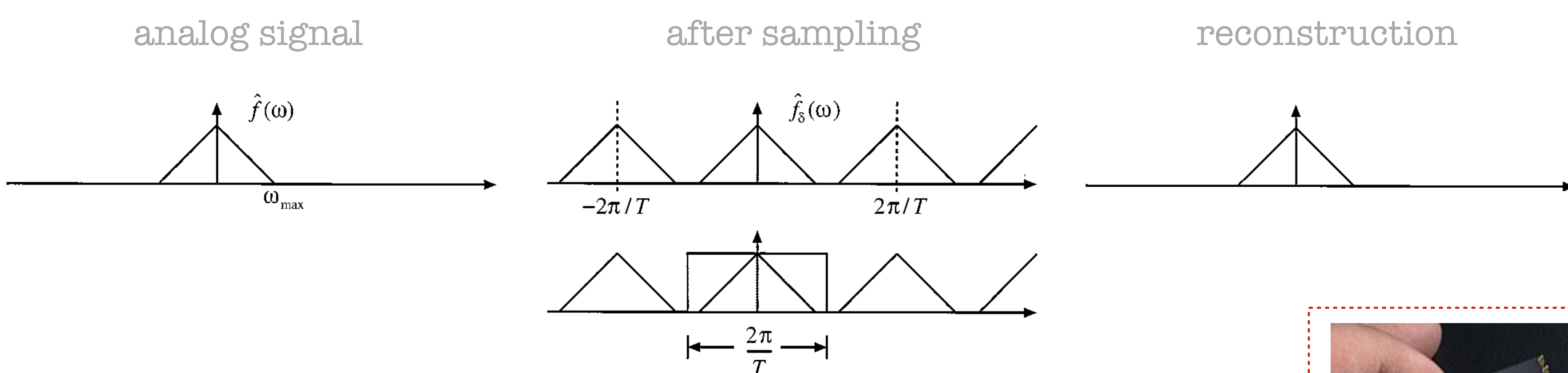
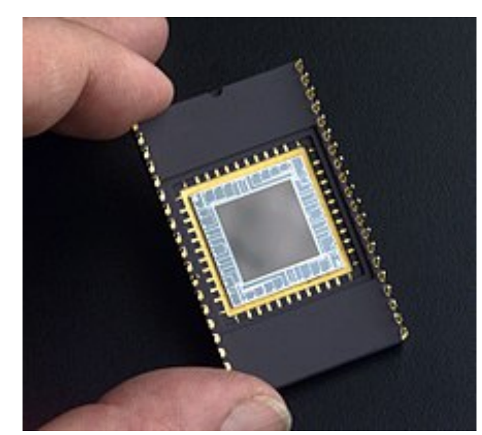


Sampling dictates the way we design sensing systems



We are building sensors that acquire massive quantities of imaging data

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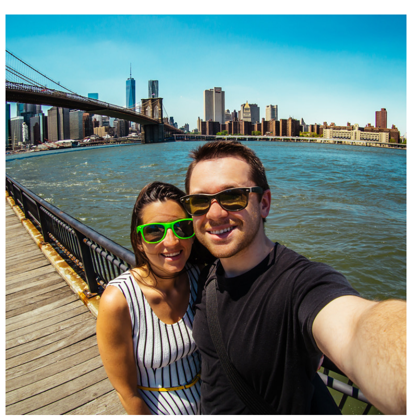
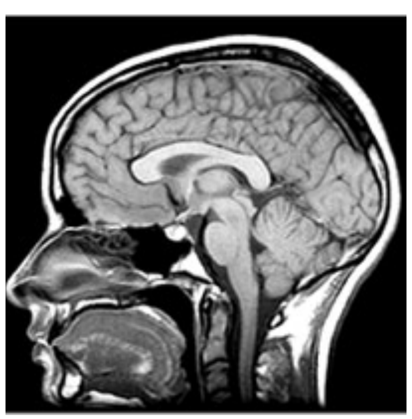
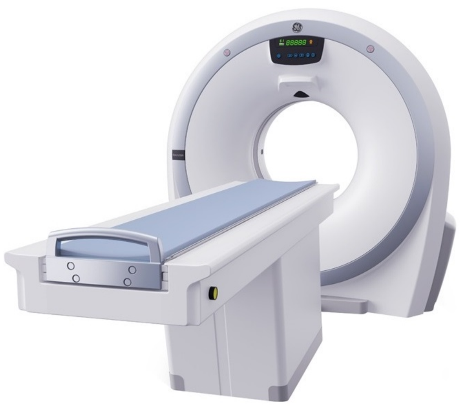
iPhone

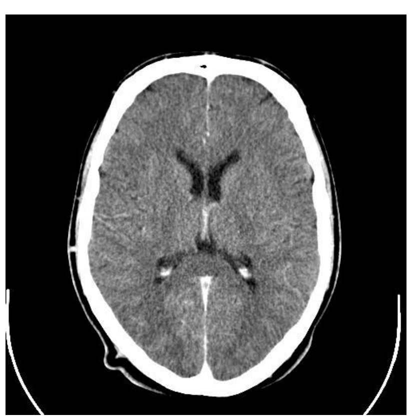


MRI

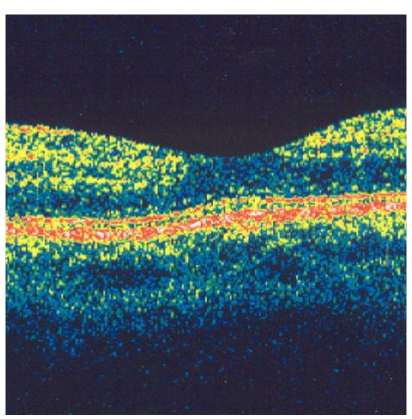


CT





OCT



We also know that visual data is highly redundant

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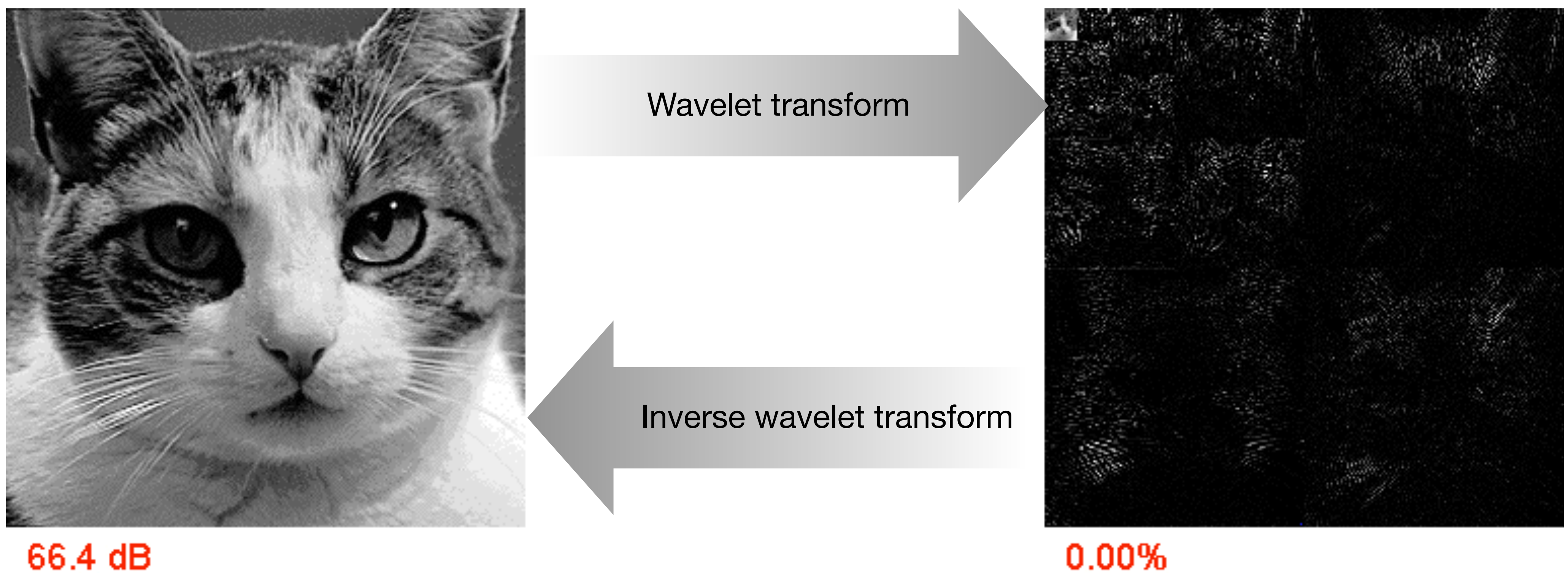

No significant visual quality loss after discarding 90% of coefficients

Contemporary imaging paradox: we collect a lot of **redundant** data

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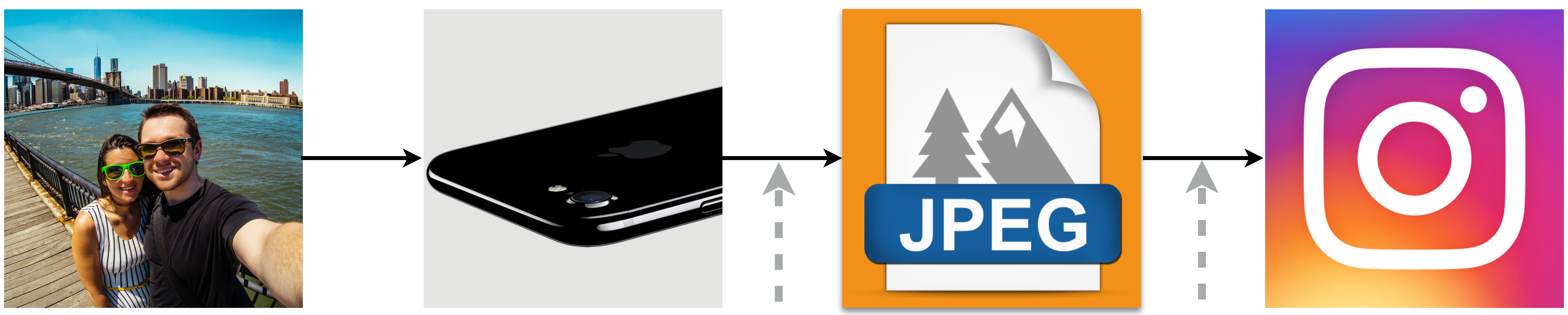
Contemporary imaging paradox: we collect a lot of **redundant** data

A long established sensing pipeline in imaging



True object

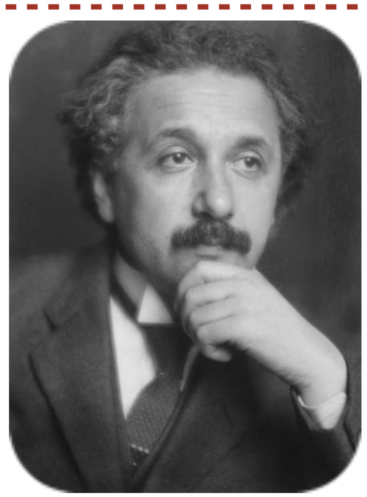
Measure

Compress

Use

N pixels

K coefficients
(K << N)



Is there a smarter way?

Compressive imaging makes us rethink the design of traditional imaging pipelines

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Acquire compressive measurements
with less samples than Shannon

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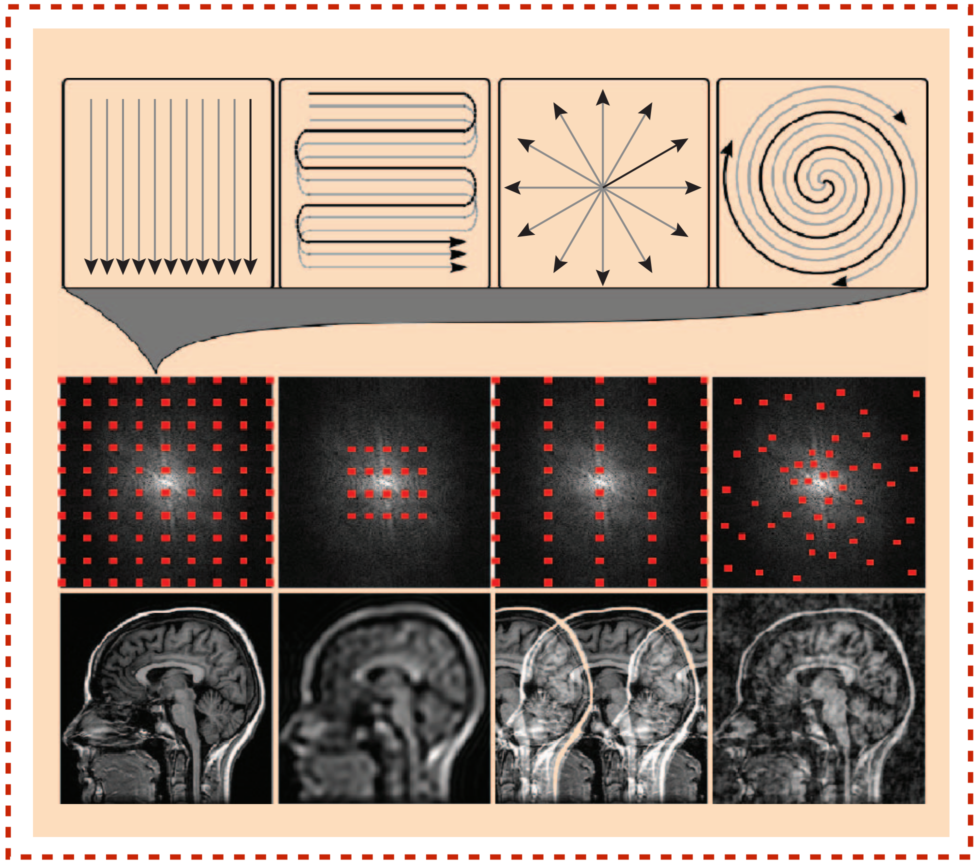

With traditional reconstruction
dropping samples would result
in visible artifacts in the image

Lustig *et al.*, *Magnetic Resonance in Medicine*, 2007

Compressive imaging makes us rethink the design of traditional imaging pipelines

Acquire compressive measurements with less samples than Shannon

Formulate reconstruction as a nonlinear optimization problem

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Formulate reconstruction as a nonlinear optimization problem

$$\boxed{\min_{\mathbf{f}} \left\{ \|\mathbf{y} - \mathbf{H}\mathbf{f}\|_{\ell_2}^2 \right\} \quad \text{s.t.} \quad \|\mathbf{W}\mathbf{f}\|_{\ell_0} \leq K}$$

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Code the optimization algorithm and solve on a computer

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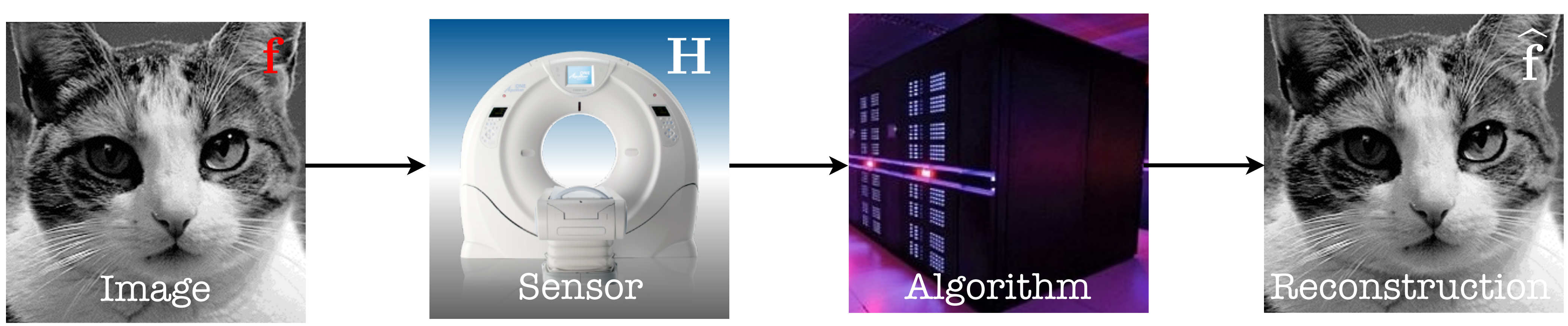

Optimization for improving the resolution of a laser sensor

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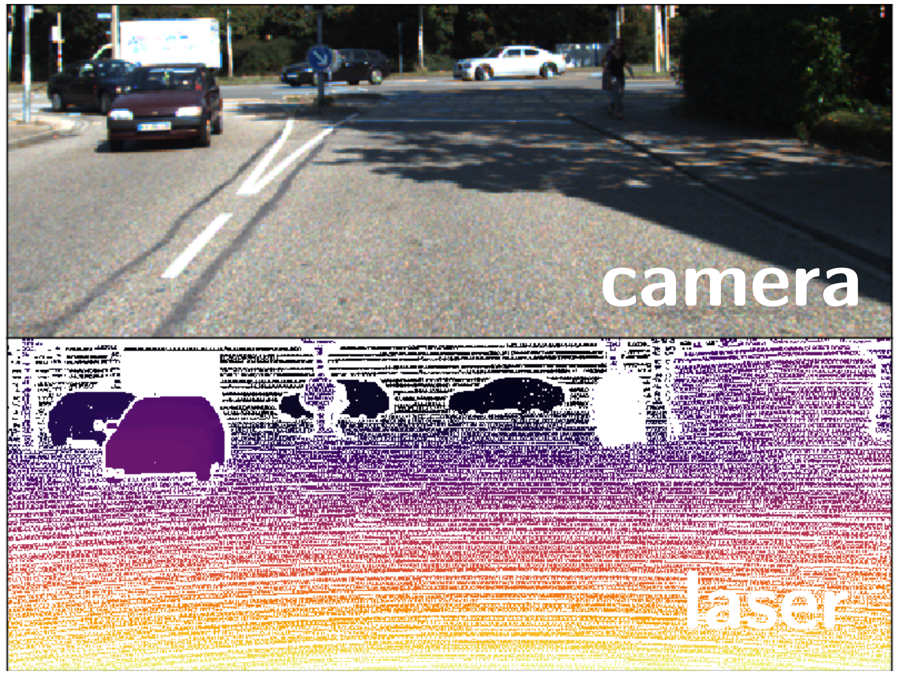
Optimization for improving the resolution of a laser sensor

Laser sensors have low-spatial resolution



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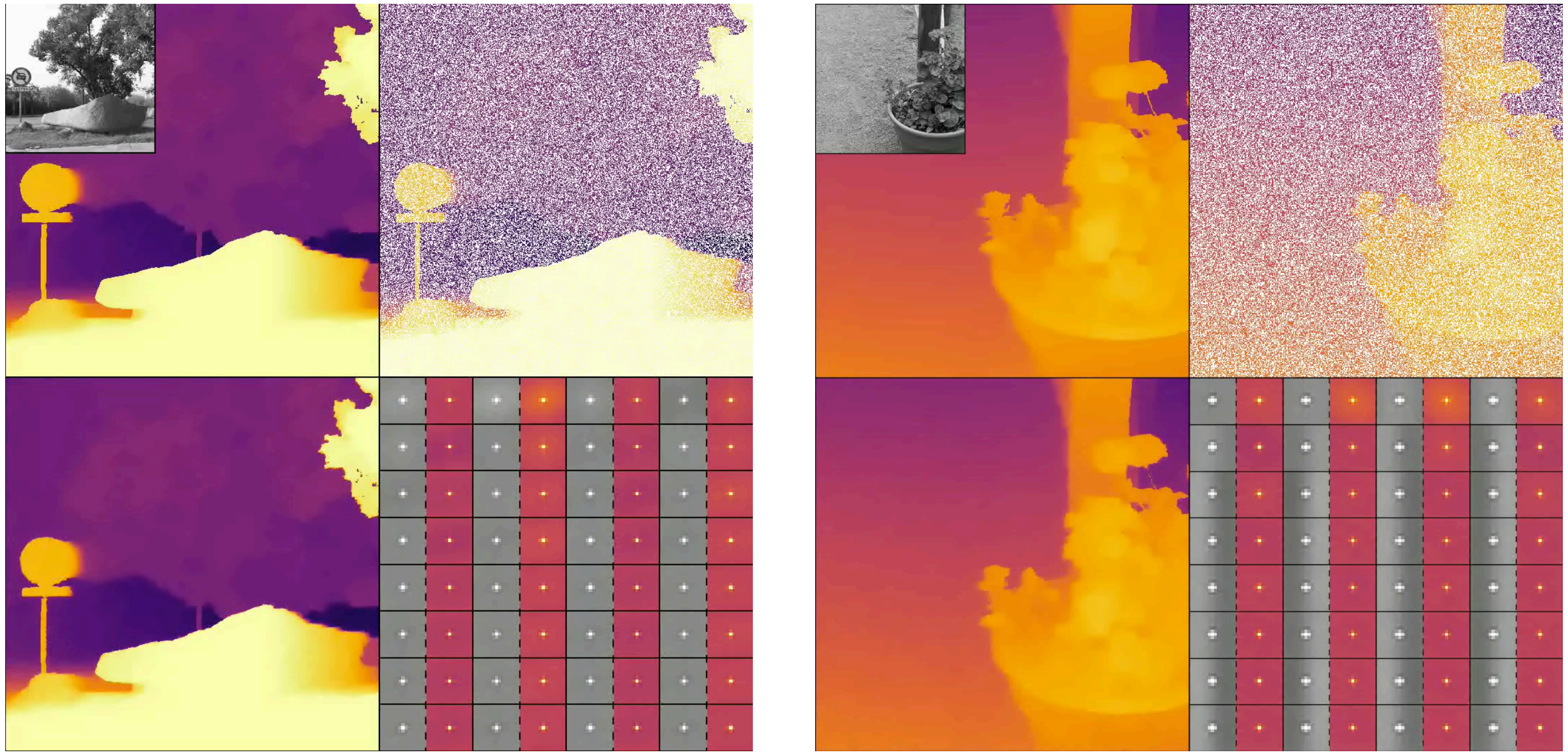
Laser sensors have low-spatial resolution

One can use write an optimization problem to enhance the resolution by fusing camera and laser



Degraux, Kamilov, Boufounos, Liu, *Proc. ICIP 2016*

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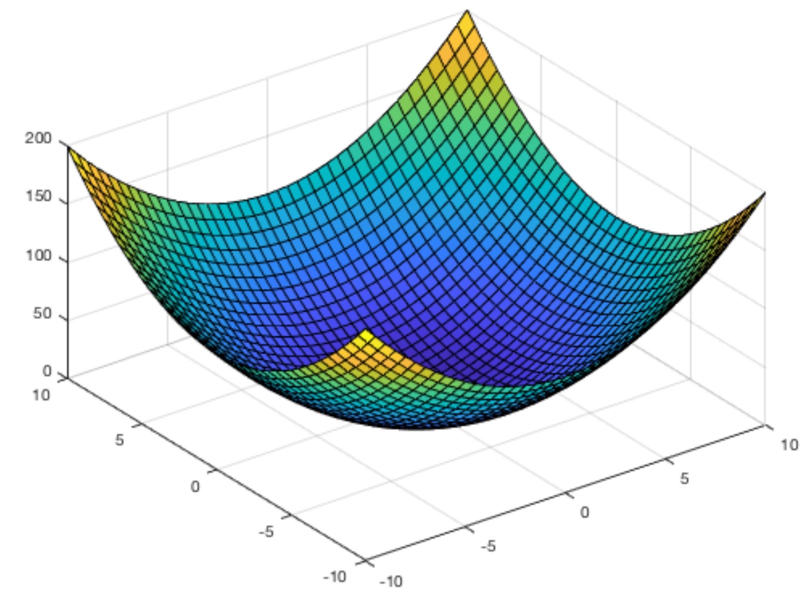
The goal of optimization is to find the smallest value of a function under constraints

Mathematical optimization problem

$$\begin{array}{ll} \text{minimize} & f(\mathbf{x}) \\ \text{subject to} & \mathbf{x} \in \mathcal{X} \end{array}$$



- Optimization variable: $\mathbf{x} = (x_1, \ldots, x_n)$
- Objective function: $f : \mathbb{R}^n \to \mathbb{R}$
- Contraint set: $\mathcal{X} = \{\mathbf{x} \,|\, \mathbf{h}(\mathbf{x}) = 0\} \cap \{\mathbf{x} \,|\, \mathbf{g}(\mathbf{x}) \leq 0\}$
- Optimal solution $\mathbf{x}^*$ has smallest value of f among all vectors that satisfy the constraints

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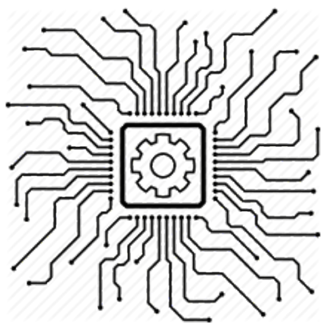
Sample applications of optimization: finance, engineering, and computer science

portfolio optimization

- variables: amounts invested in different assets
- constraints: budget, minimum return
- objective: overall risk or return variance

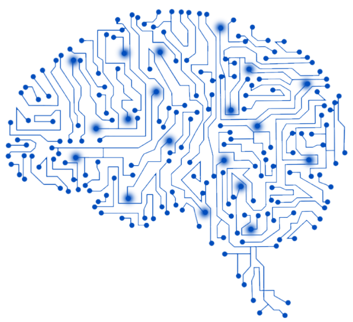
device sizing in electronic circuits

- variables: device width and lengths
- constraints: manufacturing limits, timing requirenemnts
- objective: power consumption



machine learning

- variables: model parameters
- constraints: prior information, parameter limits
- objective: prediction error



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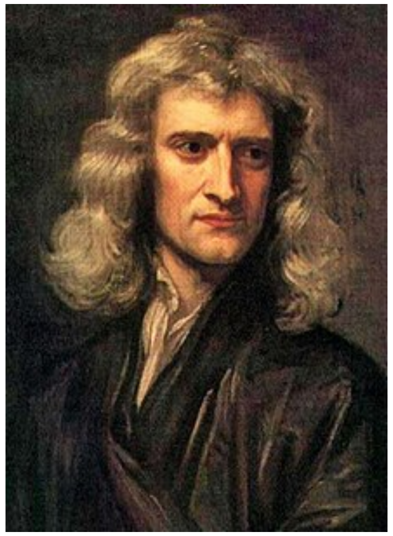
Brief history of optimization

Antiquity: Greek mathematicians solve some optimization problems that are related to their geometrical studies

- Euclid (300bc): minimal distance between a point and a line
- Heron (100 bc): light travels between two points through the path with shortest length when reflecting from a mirror

17th-18th: before the invention of calculus of variations only some separate optimization problems are being investigated

- Kepler (1615): Optimal dimensions of wine barrel. Formulated the secretary problem when searching for a new wife
- Fermat: derivative of a function vanishes at the extreme point the (1636). Shows that light travels between two points in minimal time (1657)



Newton (1660s) and Leibniz (1670s) create mathematical analysis that forms the basis of calculus of variations

http://www.mitrikitti.fi/opthist.html

Brief history of optimization

19th century: the first optimization algorithms are presented

- Legendre (1806): least square method, which also Gauss claims to have invented
- Cauchy (1847): presents the gradient method
- Gibbs shows that chemical equilibrium is an energy minimum

20th century: the field of algorithmic research expands as electronic calculation develops

- von Neuman and Morgenstern (1944): dynamic programming for solving sequential decision problems
- Dantzig (1947): simplex method for solving LP-problems
- Kuhn and Tucker (1951): reinvent optimality conditions for nonlinear problems. Similar conditions in 1939 by Karush.
- Nesterov (1983): accelerated gradient method
- Karmarkar (1984): polynomial time algorithm for LP-problems begins a boom of interior point methods.
- Modern era: nonsmooth analysis, stochastic optimization…

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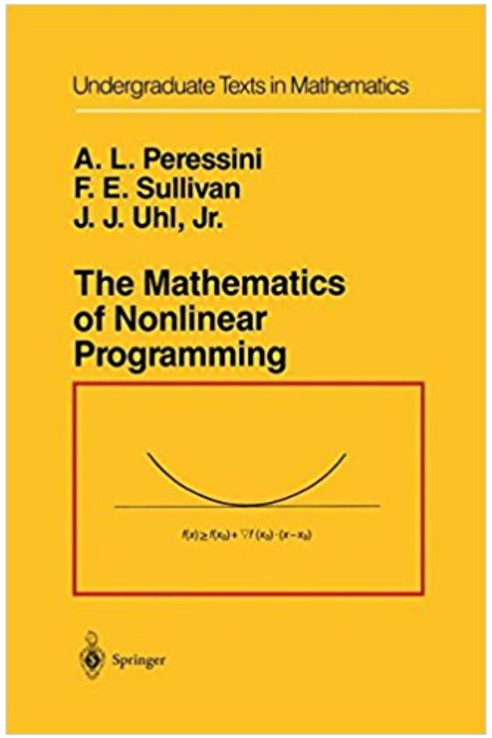
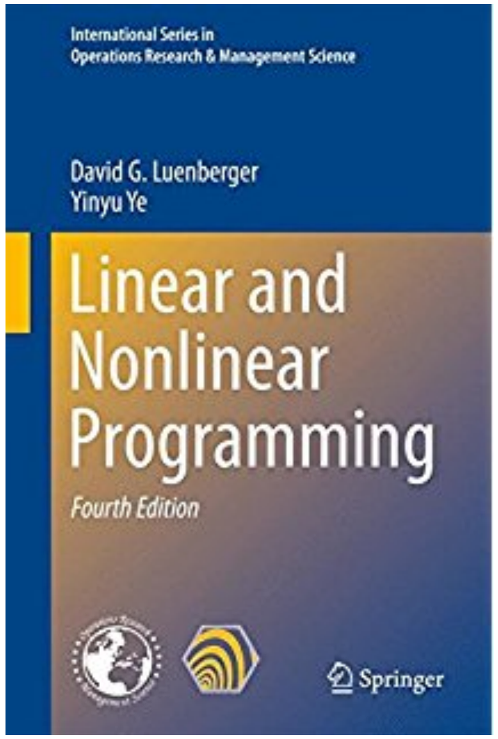
Practical information

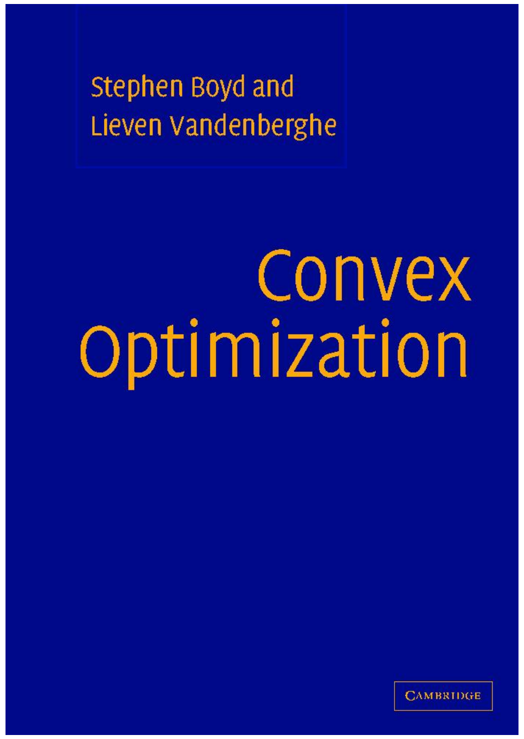
Course website:
https://cigroup.wustl.edu/teaching/ese415

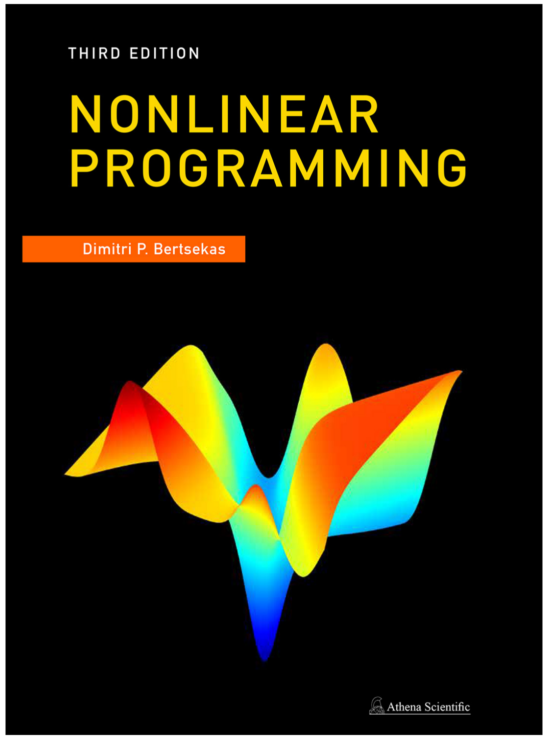
Schedule:
Lectures: Tue and Thu at 1:00-2:30 pm
Recitations: ???

Recommended reading:









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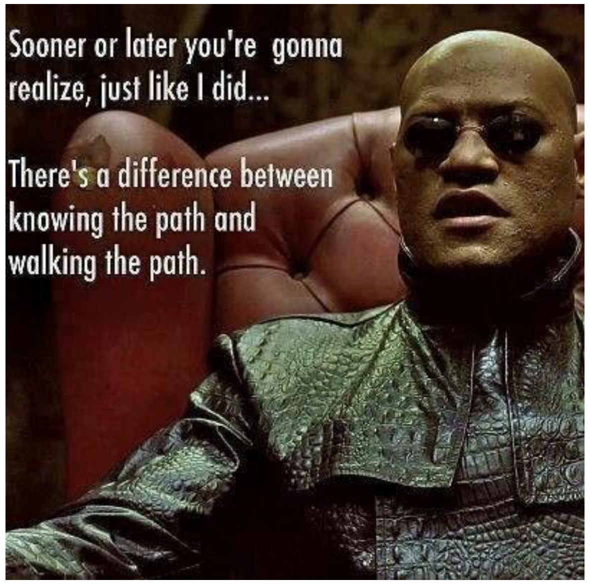
Practical information

By the end of the semester, hopefully, you will be able to:
- recognize and formulate problems as optimization
- characterize optimal solutions
- develop code for optimization algorithms

Topics:
- optimality conditions
- convex sets and functions
- constrained and unconstrained optimization
- optimization algorithms
- analysis of optimization algorithms
- examples and applications



Practical information

Grading policy:

- homework (40%) consists of 6 assignments
- midterm (30%) is on Thursday, 8 March 2017
- final (30%): is on Tuesday, 8 May 2017

Grading convention:

- 93-100 A
- 90-92 A-
- 88-89 B+
- 83-87 B
- 80-82 B-
- 70-79 C
- 60-69 D
- below 60 F

Practical information


Head TAs:

- Hesam Mazidi (hmazidi@wustl.edu)
- Yu Sun (sun.yu@wustl.edu)

TA team:

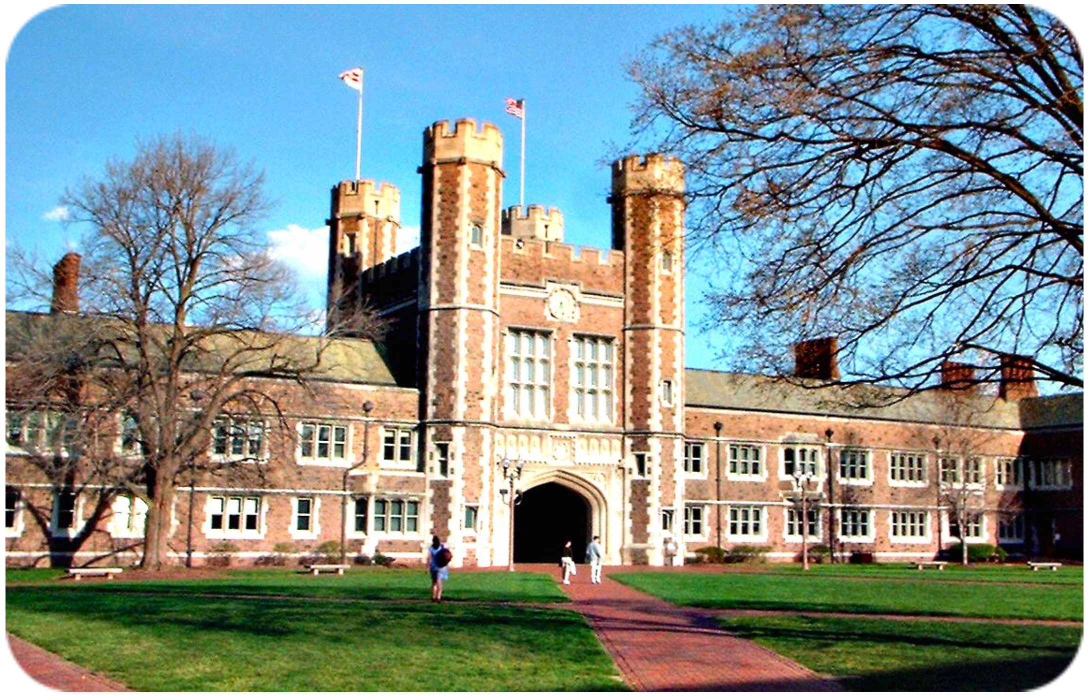
- Tao Ge (getao@wustl.edu)
- Yueying He (he.yueying@wustl.edu)
- Yunshen Huang (huang.yunshen@wustl.edu)
- Xin Ning (xin.ning@wustl.edu)
- Chang Xue (chang.xue@wustl.edu)
- Boyang Zhou (bzhou24@wustl.edu)
- Yufei Zhou (yufei.zhou@wustl.edu)

Conclusion

Optimization is extensively used in almost all engineering applications

The goal of ESE 415 is to help you understand and apply the basics

Optimization is still an active research area with many open questions



CONTACT INFO

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